

# Term Structure Analysis of Option Implied Volatility in the Brazilian Market

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## Abstract

This paper aims at predicting the volatility term structure of a given asset. The model is based on the GARCH modeling of the asset's volatility, from which the term structure is derived. We test if the model is able to accommodate the term structure response to volatility shocks. Using data from two important Brazilian companies, the model indeed improved standard predictions for the volatility term structure by relating the size of the volatility shock to the maturity of the option used to estimate the asset's volatility.

**Keywords:** Volatility term structure, Volatility shocks, Brazilian options market

## 1 Introduction

Volatility has become an object of great importance in financial markets over the last decades. While for casual investors volatility seems like a problem to

be avoided, the more experienced traders have to understand its dynamics and use it in their favor to compose their investment portfolios. Thus, rather than deploring higher levels of volatility, those who deal with derivative securities work to understand it more precisely, forecast it more accurately and correctly manage their portfolio exposure.

Successfully, Engle (1982) and Bollerslev (1986) introduced autorregressive conditional heteroskedastic (ARCH) and generalised ARCH (GARCH) models, which allow for a time-varying conditional variance and are able to accommodate volatility clustering and leptokurtosis. After that, many extensions have been proposed to better explain specific phenomena (like, for example, the leverage effect), but one must always keep in mind the tradeoff between better (in-sample) fitting and robust (out-of-sample) prediction. The debate on this topic is still very strong but there is no clear support that these more complex models are efficient when compared with GARCH models.

The present study aims at testing the continuous-time volatility model proposed by Hull (2012). The model was, to the best of our knowledge, never tested. We investigate whether the model is capable of capturing the impact of changes in the instantaneous volatility on the volatility term structure: We will argue that the model is able to adjust shocks in the underlying asset's instantaneous volatility as a function of its options maturities.

## **2 Literature Review**

Adding on the work of Bollerslev (1986), alternative models have been proposed to address some arising issues. For example, Nelson (1991) develops an extension to GARCH models which addresses potential weaknesses of the original GARCH model such as the assumption that positive and negative residuals are symmetrical and the nonnegativity constraints on the estimated parameters. Engle and Ng (1993) examine the Japanese stock returns and find that negative shocks introduce more volatility than positive shocks. The authors fit a series of models to try to capture this asymmetry and the GJR model proposed by Glosten, Jagannathan and Runkle (1993) brings the best results. Campbell and Hentschel (1992) modify the GARCH model to allow for an asymmetric model that amplifies large negative returns and dampens large positive returns, making stock returns negatively skewed and increasing the potential for large crashes. Noh, Engle and Kane (1994) extend the GARCH specification to include weekend effects, Heynen, Kemma and Vorst (1994) analyse the relation between short- and long-term volatilities in the European Option Exchange and Amsterdam Stock Exchange using mean-reverting, GARCH and EGARCH models. Duan (1995), Stentoft (2005), Sabbatini and Linton (1998) and Duan and Zhang (2001) are more examples to study volatility processes in different markets.

In the Brazilian market, Almeida and Dana (2005) aim at identifying the main characteristics of the volatility process which drive a typical stock in the Brazilian market but apply a formulation of a stochastic volatility model proposed earlier in the literature. Mota and Fernandes (2004) examine the volatility of the Ibovespa stock market index and compare its estimation by different methods, including GARCH models. Gabe and Portugal (2003) apply implied and statistic volatilities to discuss the volatility forecasting power using Telemar S.A. options, a Brazilian telecommunications company traded in Brazilian stock market. Morais and Portugal (1999) compare the forecasting performance of deterministic and stochastic volatility modeling, based on the Ibovespa stock market index. They tested deterministic models including ARCH, GARCH, GARCH-t, EGARCH and GJR. The study concludes that both deterministic and stochastic approaches perform quite well.

None of the aforementioned articles focuses on applying a continuous-time GARCH model for option volatility term-structure, let alone analyzing how volatility shocks are transferred to the term structure. Therefore, we believe that the present work can add to the literature by examining the application of the model in an important emerging market economy. The contribution of this paper is to test a never-before-tested model that forecasts the way in which the volatility term structure responds to instantaneous volatility shocks.

### 3 Theoretical Framework

#### 3.1 High-Frequency Data and Realised Volatility

The availability of high-frequency data has allows for intraday volatility estimation as a proxy for the realised volatility. Let  $r_n$  be the daily log return at day  $n$  and that this day is divided in  $m$  periods (of, say, 10 minutes). The daily log return based on the log return of each period can be expressed as:

$$r_n = \sum_{i=1}^m r_{n,i} \quad (1)$$

Assuming  $r_{n,i}$  is a white noise series then:

$$\sigma_n^2 = m \text{Var}(r_{n,1}) \quad (2)$$

where  $\text{Var}(r_{n,1})$  can be estimated by:

$$\text{Var}(r_{n,1}) = \frac{1}{m} \sum_{i=1}^m r_{n,i}^2 \quad (3)$$

Putting it all together, we can have an estimation of the realised daily variance as follows:

$$\hat{\sigma}_n^2 = \sum_{i=1}^m r_{n,i}^2 \quad (4)$$

Apart from mathematical developments, it is preferred to work with volatility instead of variance, since it can be easily compared to log-returns. The estimated realised volatility (square-root of this realised variance) is important because it can be used as a benchmark to compare volatility models, since, unlike realised returns, true realised volatilities are not observable.

### 3.2 A Continuous-Time GARCH Model for Volatility Term Structure

In this section the model analyzed by this study for predicting volatility is presented. The predictions are used to build the volatility term structure. The model is developed based on a GARCH(1,1) process, taken from Hull (2012). We show and explain in section 5.1 that the GARCH(1,1) model is the preferred one to model the conditional volatility of the time series used by this study. Empirically, GARCH models with higher lags are extremely rare in the related literature.

At the end of day  $n - 1$ , the estimated variance rate for day  $n$  using GARCH(1,1) is given by:

$$\sigma_n^2 = (1 - \alpha - \beta) \sigma_\infty^2 + \alpha r_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (5)$$

that can also be written as follows:

$$\sigma_n^2 - \sigma_\infty^2 = \alpha (r_{n-1}^2 - \sigma_\infty^2) + \beta (\sigma_{n-1}^2 - \sigma_\infty^2) \quad (6)$$

Consequently, if the variance rate for day  $n + t$  is to be estimated, the following equation can be employed:

$$\sigma_{n+t}^2 - \sigma_\infty^2 = \alpha (r_{n+t-1}^2 - \sigma_\infty^2) + \beta (\sigma_{n+t-1}^2 - \sigma_\infty^2) \quad (7)$$

Since the expected value of  $r_{n+t-1}^2$  at the end of day  $n + t - 2$  is  $\sigma_{n+t-1}^2$ , we can make use of the law of iterated expectations and write:

$$E_{n-1} [\sigma_{n+t}^2 - \sigma_\infty^2] = (\alpha + \beta) E_{n-1} [\sigma_{n+t-1}^2 - \sigma_\infty^2] \quad (8)$$

If this equation is used repeatedly  $(\alpha + \beta)$  must be powered to  $t$ :

$$E [\sigma_{n+t}^2] = \sigma_\infty^2 + (\alpha + \beta)^t (\sigma_n^2 - \sigma_\infty^2) \quad (9)$$

Thus, equation (9) forecasts the volatility on day  $n + t$  based on the information available on day  $t - 1$ . To construct the volatility term structure suppose it is day  $n$  and define:

$$V(t) = E(\sigma_{n+t}^2) \quad (10)$$

and

$$a = \ln \frac{1}{\alpha + \beta} \quad (11)$$

such that equation (9) becomes:

$$V(t) = \sigma_\infty^2 + e^{-at} [V(0) - \sigma_\infty^2] \quad (12)$$

Note that  $V(t)$  is an estimate of the variance rate in  $t$  days. Equation (13) gives the average variance rate per day between times 0 and  $T$ :

$$\frac{1}{T} \int_0^T V(t) dt = \sigma_\infty^2 + \frac{1 - e^{-aT}}{aT} [V(0) - \sigma_\infty^2] \quad (13)$$

Thus,  $\sigma(T)^2$  can be used to represent the volatility per annum that should be used to price a  $T$ -day option under GARCH(1,1):

$$\sigma(T)^2 = 252 \left( \sigma_\infty^2 + \frac{1 - e^{-aT}}{aT} [V(0) - \sigma_\infty^2] \right) \quad (14)$$

Note that (14) assumes 252 trading days per year, such that  $\sigma(T)^2$  is 252 times the average variance rate per day. To better understand the impact of changes in volatility, (14) can be written as:

$$\sigma(T)^2 = 252 \left[ \sigma_\infty^2 + \frac{1 - e^{-aT}}{aT} \left( \frac{\sigma(0)^2}{252} - \sigma_\infty^2 \right) \right] \quad (15)$$

such that when  $\sigma(0)$  changes by  $\Delta\sigma(0)$ ,  $\sigma(T)$  will approximately change by:

$$\Delta\sigma(T) \approx \frac{1 - e^{-aT}}{aT} \frac{\sigma(0)}{\sigma(T)} \Delta\sigma(0) \quad (16)$$

Note that, from equation (14), the mean volatility until the maturity of the option, i.e.,  $\sigma(T)^2$ , is a function of the unconditional variance  $\sigma_\infty^2$ , the time to maturity  $T$ , the parameters of the GARCH process represented by  $a$  and the instantaneous volatility  $\sigma(0)$ . Note also that the term  $(1 - e^{-aT})/aT$  is always positive and will decrease when  $T$  increases, since  $a$  is constant after the GARCH parameters have been determined. Therefore, the further away the expiration date of the option is, the smaller the deviation from the unconditional volatility will be for the mean volatility up to this maturity. Moreover,

the model proposes a reversion to the unconditional volatility over time and the speed of this reversion is dictated by  $a = \ln(1/(\alpha + \beta))$ , where  $a > 0$  always, since the GARCH modelling implies that  $\alpha + \beta < 1$  for a stable GARCH process. Higher  $a$  values imply faster reversion to the unconditional volatility, while small  $a$  values result in slower reversion.

Equation (16) focuses solely on the impact of changes in the instantaneous volatility on the expected mean volatility of the underlying asset until the desired maturity. The first term,  $(1 - e^{-aT})/aT$ , which is also present in equation (14), serves as a smoother of the instantaneous volatility shock. As mentioned above, this term is smaller the further away we are from the expiration date. Thus, the model proposes that shocks in today's volatility have small impact on options whose maturities are far in the future, and large impact on options that expire in the near future: This is a clear consequence of the mean-reverting nature of the GARCH model.

The relation between instantaneous volatility and mean volatility until expiration is represented by the term  $\sigma(0)/\sigma(T)$ . When the instantaneous volatility is higher than the expected mean volatility until maturity, the shock in instantaneous volatility is amplified, since  $\sigma(0)/\sigma(T) > 1$ . The opposite,  $\sigma(0)/\sigma(T) < 1$ , means that the expected volatility level until maturity is higher than the current one and thus shocks on instantaneous volatility have smaller impact on the mean volatility.

The last term, the shock itself ( $\Delta\sigma(0)$ ), is the only term that indicates the direction of the movement. If the shock on instantaneous volatility is positive, the model proposes that the shock on the expected mean volatility until maturity will also be positive. The reciprocal reasoning is valid for negative shocks.

## 4 Methodology and Data

We had decided to focus on Brazil, a doubtless important emerging economy that still misses important studies regarding volatility processes. The analysis will be entirely undertaken based on data collected from market databases (primarily Bloomberg and Economatica). The data will be broken in in-sample and out-of-sample intervals. The in-sample will be used to estimate the GARCH model whose parameters are necessary for the construction of the volatility term structure under the model analyzed. For the estimation of the parameters of the GARCH process, daily log-return data will be used, calculated from daily closing prices for the selected stocks in the period. The best model will be selected based on the Akaike information criterion and on the significance of the parameters.

We then verify if the model can capture the impact of changes in the instantaneous volatility. The difference between two following days volatilities is

considered as a shock. In this approach, the increase or decrease in the instantaneous volatility from one day to another is an input for the model tested, which indicates the impact of that change in the volatility term structure, as captured by option implied volatilities.

The selected financial series for the study are Vale S.A. and Petrobras S.A. preferred stocks traded in the Brazilian stock market under the tickers VALE5 and PETR4 respectively. Both stocks were chosen not only because of their extreme importance on the Brazilian economic scenario but also because their market liquidity (especially their options' liquidity). VALE5 and PETR4 stocks and options are amongst the most traded securities in the Brazilian BM&FBovespa exchange. From January 1999 to March 2014, VALE5 and PETR4 stocks corresponded in average for 19% of the total monthly trade volume in the BOVESPA stock market. The liquidity of their options is even more relevant in the Brazilian derivatives market: on a typical day, the options on both stocks sum up to more than 90% of the traded volume (as according to market data in 2014).

The selected time frame for the daily closing stock prices begins in January 1999, when the Brazilian Central Bank decreed that the exchange rate between Brazilian Real and US Dollars would follow a floating regime (before that date, the exchange rate followed an upper limit system) and ends in April 2014. The augmented Dickey-Fuller test for unit-roots was able to reject the null hypothesis even at a very low significance level of 0.01% for both series, providing evidence that the return data is stationary. The stationarity of the data is important when working with GARCH models.

To calculate the realised volatility, intraday data has been gathered. The frequency of the data used to calculate realised volatility is a discussion in the literature, since too short intervals between returns can result in microstructure problems. Andersen et al. (2001) propose a five minute interval, while Oomen (2001) proposes a 25 minute interval and Giot and Laurent (2001) argue in favor of 15 minute data. However, an interval equal or greater than five minutes is considered to eliminate most of the microstructure problems. Thus, considering this recommendation and also the availability of data, 10-minute data has been gathered for 150 days between August 20th 2013 and April 2nd 2014, totalizing 6,450 observations for both VALE5 and PETR4, corresponding to 43 observations each day. The data was collected from the Bloomberg database. The return between the last price of a day and the opening price of the following day was included in the calculation of realised volatilities to account for the overnight volatility.

## 5 Results

### 5.1 Estimation of the GARCH Model

The best fitting models for the VALE5 and PETR4 log-return series was the GARCH(1,1), as presented in tables 1 and 2. These tables present the Akaike information criterion (AIC) for the models, as well as significant and non-significant parameters and the p-value for the Ljung-Box LB statistic of lag 10 for the squared residuals. A p-value greater than 0.05 indicates no evidence of autocorrelation at 5% significance.

**Table 1:** VALE5 GARCH Estimated Models.

Model	AIC	Significant Parameters	Non-Significant Parameters	LB(10) p-value
ARCH(1)	-4.6949	$\alpha_0, \alpha_1$	-	0.0000
ARCH(2)	-4.7476	$\alpha_0, \alpha_1, \alpha_2$	-	0.0000
GARCH(1,1)	-4.8248	$\alpha_0, \alpha_1, \beta_1$	-	0.5471
GARCH(1,2)	-4.8246	$\alpha_0, \alpha_1, \beta_1$	$\alpha_2$	0.5693
GARCH(2,1)	-4.8246	$\alpha_0, \alpha_1, \beta_1$	$\beta_2$	0.5587
GARCH(2,2)	NA	No convergence	NA	NA

**Table 2:** PETR4 GARCH Estimated Models.

Model	AIC	Significant Parameters	Non-Significant Parameters	LB(10) p-value
ARCH(1)	-4.7035	$\alpha_0, \alpha_1$	-	0.0000
ARCH(2)	-4.7665	$\alpha_0, \alpha_1, \alpha_2$	-	0.0000
GARCH(1,1)	-4.8405	$\alpha_0, \alpha_1, \beta_1$	-	0.2323
GARCH(1,2)	-4.8404	$\alpha_0, \alpha_1, \beta_1$	$\alpha_2$	0.3613
GARCH(2,1)	-4.8401	$\alpha_0, \alpha_1, \beta_1$	$\beta_2$	0.2366
GARCH(2,2)	-4.8400	$\alpha_0, \alpha_1, \beta_1$	$\alpha_2, \beta_2$	0.3994

This result comes with no surprise, since it has been well documented in the literature that the GARCH(1,1) model has shown consistent results in modelling financial time series, as observed in Cao and Tsay (1992), Sabbatini and Linton (1998) and Duan and Zhang (2001), just to cite a few of them. Table 3 presents the estimated parameters for the GARCH(1,1) models for VALE5 and PETR4.

**Table 3:** VALE5 GARCH(1,1) Estimated Parameters.

Parameter	Value	Std. Error	p-value	Parameter	Value	Std. Error	p-value
$\alpha_0(VALE5)$	$1.431 \times 10^{-5}$	$3.426 \times 10^{-6}$	0.0000	$\alpha_0(PETR4)$	$1.373 \times 10^{-5}$	$3.007 \times 10^{-6}$	0.0000
$\alpha_1(VALE5)$	$7.678 \times 10^{-2}$	$1.130 \times 10^{-2}$	0.0000	$\alpha_1(PETR4)$	$7.619 \times 10^{-2}$	$9.377 \times 10^{-3}$	0.0000
$\beta_1(VALE5)$	$8.960 \times 10^{-1}$	$1.573 \times 10^{-2}$	0.0000	$\beta_1(PETR4)$	$8.974 \times 10^{-1}$	$1.280 \times 10^{-2}$	0.0000



## 5.2 Adjusting the Volatilities and Modelling the Shocks

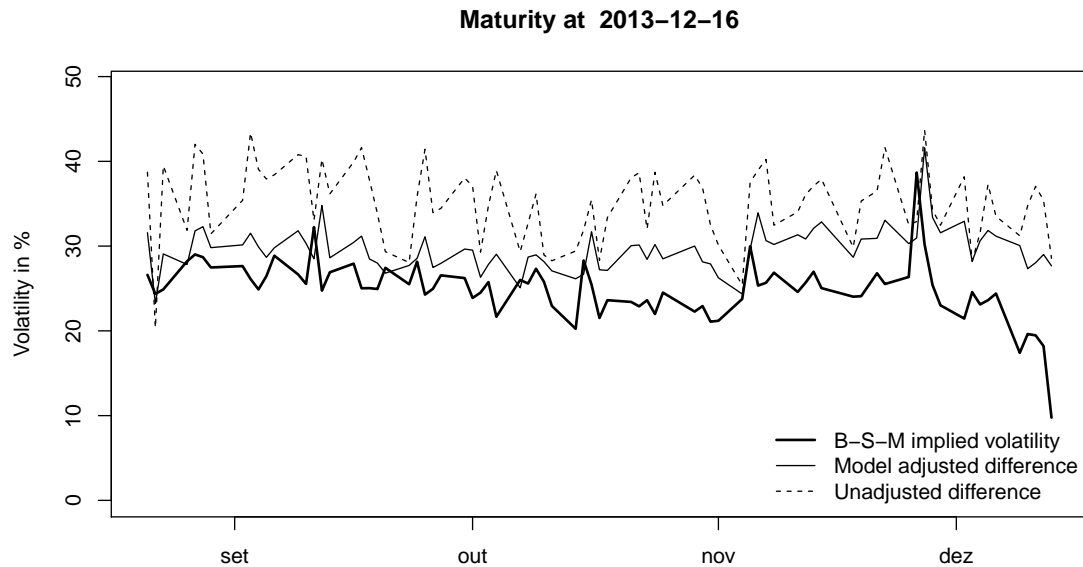
As suggested in Hull (2012), the model presented in section 3.2 would be able to predict the impact of shocks in volatility. Hence, equation (16) should bring satisfactory results. Thus, we analyze how (16) works with the data used by this study.

Our approach uses equation (16) to predict tomorrow's volatility considering  $\sigma(T)$  as current Black-Scholes-Merton implied volatility,  $\sigma(0)$  as realised volatility and  $\Delta(\sigma(0))$  as the difference between tomorrow's GARCH predicted volatility and today's realised volatility. Notice that if this approach works reasonably well, traders can use the methodology to forecast tomorrow's volatility, implying successful trades (for example, in the options or buying and selling volatility through an accordingly portfolio strategy).

Considering that there is a change in instantaneous volatility of the underlying, it is natural to imagine that this change will influence the price of the options on that underlying. It is, however, naive to admit that all options will be impacted in the same way. Intuitively, options that expire closer to present day will be more influenced by this change in instantaneous volatility than option that expire further in the future. This behavior is what is represented in equation (16). That way, the model should be able to improve a certain prediction on future volatility by adjusting that prediction with the time to expiration of a given option. To test this assumption, GARCH estimated volatility for the next day is considered the benchmark forecast. In other words, the best guess for tomorrow's volatility would be today's end of the day Black-Scholes-Merton volatility plus the difference between tomorrow's GARCH estimated volatility and that Black-Scholes-Merton volatility. Our approach is more refined and states that  $\Delta(\sigma)$  can be adjusted by equation (16) to reflect the model adjusted difference in volatility between days  $t$  and  $t + 1$ . Thus, this approach basically uses a "raw" GARCH prediction for the following day and adjusts its difference in relation to the present Black-Scholes-Merton volatility according to the expiration date of a given option.

Figures 1 and 2 present the results for VALE5 and PETR4. Tables 4 and 5 present the error measures. All error measures are lower for all adjusted forecasts, indicating that the model is in fact able to improve the GARCH predictions. The charts for VALE5 and PETR4 for all selected months are readily available from the authors.

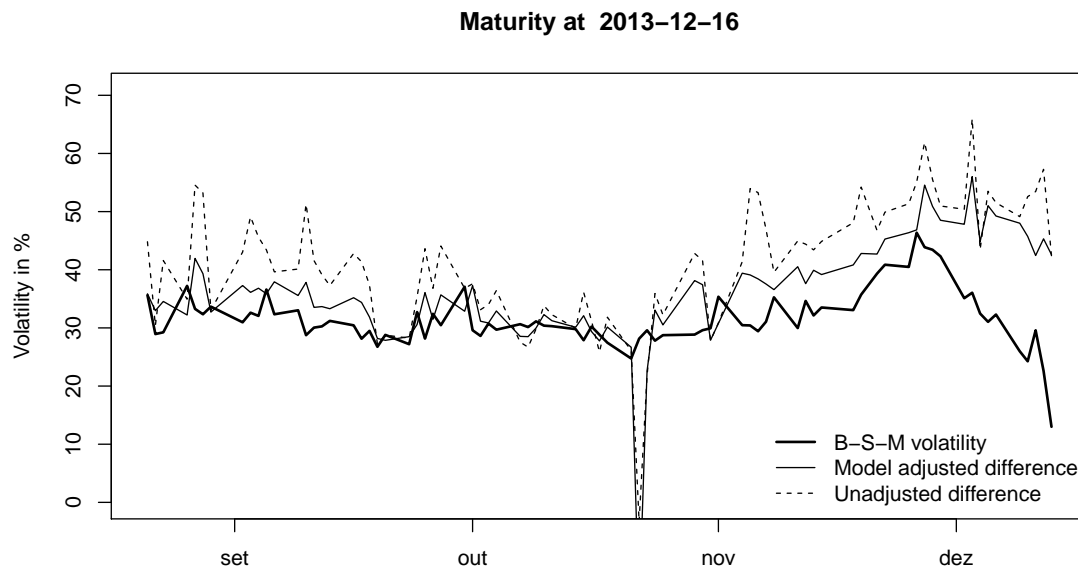
**Figure 1:** Black-Scholes-Merton, model adjusted differences and unadjusted "raw" GARCH differences for the options expiring in December 2013 for VALE5.



**Table 4:** Error measures for VALE5 options GARCH estimated volatility and GARCH adjusted shocks. The benchmark measure is the Black-Scholes-Merton implied volatility for options expiring from September 2013 to March 2014. RMSE stands for Root Mean Squared Error, MAD is the Mean Absolute Deviation and MAPE is the Mean Absolute Percentage Error.

Original GARCH shock	Sep-13	Oct-13	Nov-13	Dec-13	Jan-14	Feb-14	Mar-14
RMSE	12.82%	10.68%	10.96%	11.33%	10.85%	11.05%	10.12%
MAD	11.51%	9.79%	10.05%	10.39%	10.07%	9.85%	9.28%
MAPE	52.65%	41.64%	41.11%	44.26%	42.32%	41.23%	35.87%
Model adjusted shocks	Sep-13	Oct-13	Nov-13	Dec-13	Jan-14	Feb-14	Mar-14
RMSE	8.92%	6.12%	5.72%	6.01%	5.13%	5.50%	4.39%
MAD	7.70%	5.24%	4.95%	5.09%	4.29%	4.36%	3.69%
MAPE	37.53%	24.05%	20.95%	22.68%	19.35%	19.14%	14.36%

**Figure 2:** Black-Scholes-Merton, model adjusted differences and unadjusted "raw" GARCH differences for the options expiring in December 2013 for PETRA4.



**Table 5:** Error measures for PETRA4 options GARCH estimated volatility and GARCH adjusted shocks. The benchmark measure is the Black-Scholes-Merton implied volatility for options expiring from September 2013 to March 2014. RMSE stands for Root Mean Squared Error, MAD is the Mean Absolute Deviation and MAPE is the Mean Absolute Percentage Error.

Original GARCH shock	Sep-13	Oct-13	Nov-13	Dec-13	Jan-14	Feb-14	Mar-14
RMSE	13.20%	10.07%	11.12%	13.72%	14.81%	14.09%	13.96%
MAD	11.57%	8.33%	8.77%	10.89%	12.16%	11.81%	11.78%
MAPE	41.09%	29.28%	29.02%	36.96%	43.00%	40.66%	38.94%
Model adjusted shocks	Sep-13	Oct-13	Nov-13	Dec-13	Jan-14	Feb-14	Mar-14
RMSE	9.74%	6.26%	9.18%	10.02%	13.99%	7.39%	7.06%
MAD	8.52%	5.06%	5.61%	6.74%	7.34%	5.68%	5.30%
MAPE	32.56%	18.79%	18.99%	23.85%	27.13%	20.10%	17.75%

## 6 Conclusion

The present work applied a model to forecast option implied volatility changes in the term structure in an emerging market economy, specifically Brazilian BM&FBovespa Stock Exchange. To do so, two major Brazilian companies were selected and analyzed, based on the liquidity of their stocks and call options.

We analyzed the model proposed in Hull (2012), build on the GARCH(1,1) model, which has never been tested to the best of our knowledge. We conducted an analysis in which the pure GARCH(1,1) shocks where compared to the model adjusted shocks, calculated considering the time to expiration of each option. This analysis was especially relevant because Hull (2012) explicitly comments in favor of good results although we were not able to find any research paper to confirm this hypothesis. The results showed a significant improvement in volatility prediction, not rarely halving the original percent errors. Therefore, the model proves to be useful for option implied volatility estimation as a function of time to maturity. For example, not only traders (i.e., speculators) could benefit from the model but also risk managers, since they will be able to predict their risk exposure better, e.g., calculating a more accurate VaR (value at risk).

As a limitation of the model, we recall that it assumes the data to follow a GARCH(1,1) process, which may not be the case for other assets. Moreover, the model is not straightforwardly built on GARCH extensions, such as exponential, asymmetric and threshold models. Further development of the subject could include the simulation of an hypothetical portfolio and creation of a trading strategy based on the predictions done by the model. It would then be possible to verify if the model is capable of generating profits in a real market, accounting to transaction costs.

Another approach would be to use volatility indices as benchmarks, instead of Black-Scholes-Merton implied volatility. Since the last has been shown to present limitations, the use of a well accepted index (e.g., the VIX index) could better serve as benchmark to assess the accuracy of the volatility forecasts.

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