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## On the Correct Evaluation of Cost of Capital for Project Valuation

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### Abstract

This paper fills a very important gap in the literature with a straightforward methodology that generalizes the classic Modigliani and Miller results and provides correct values for the expected return on equity and for the weighted average cost of capital (WACC). After some confusing debate in the literature, we show that these correct values make the three main project valuation approaches (WACC, flow to equity and adjusted present value) to perfectly match. Specifically, we show that the expected return on equity for finite horizon projects will be horizon dependent, which means that it will (sometimes drastically) change as the project life shortens even when all other variables remain constant (e.g., the leverage ratio). Obviously, this result will directly impact the WACC. Another interesting result is that the traditional textbook formula for the WACC is not always correct, even for perpetual projects.

**Keywords:** Cost of Capital; Project valuation; WACC; Float to equity; Adjusted present value approach; Return on equity

## 1 Introduction

It is highly surprising how practitioners are not aware of the conditions under which the classic results of [16, 17] hold<sup>1</sup>. Most importantly, in realistic projects (for example, under finite horizon) the traditional formulas for the required return on equity and, as a consequence, for the weighted average cost of capital (hereafter WACC) in a project financed (at least in part) with debt will not provide correct valuations. Therefore, managers might be forced to take wrong capital budgeting decisions when relying in misleading valuation techniques.

Although, as we will cover, academics are aware of this point, the results available in the literature are still incomplete. In this paper, we uncover important mathematical results and enrich the literature with a methodology to correctly calculate the expected return on equity and the WACC, allowing managers to correctly price projects, based on the debt repayment plan and the usual pieces of information necessary to doing so (such as the unlevered expected return on the project, the cost of debt and the debt tax shield riskness). For example, the traditional formula to the WACC (weighing the costs of equity and debt, this last one adjusted to the tax shield) is usually denoted as its definition while in fact it is not (to cite just a few, see [3, 4, 5, 12]. We show in this paper that in fact this formula is not true if part of the interests is not paid and therefore is capitalized as part of the debt principal.

In this study, we discuss the three main approaches to project valuation when using debt financing, namely the traditional WACC approach, the flow to equity approach (a.k.a. FTE) and the adjusted present value approach (a.k.a. APV). Doubtless, they must always provide the same valuation for any investment project. The issues discussed and solved here become very important because the WACC approach followed by the FTE approach seem to be the choices taken by practitioners<sup>2</sup>, even though in some cases the APV methodology should be easier and clearer.

The debate in the literature regarding generalizations of MM's traditional results is very wide. Ezzell and Miles [1, 2] main point argues that when the firm's financing strategy is to maintain a constant market value leverage ratio, the debt interest rate should be used to discount the tax shield only in period 1, while the required return on the unlevered project should be used for all other future periods. They also present a correct formula to unlever beta given

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<sup>1</sup>We will in the text refer to their classic results as MM's traditional formulas, specifically referring to the required return on equity and to the weighted average cost of capital formulas.

<sup>2</sup>[12] provides a list of studies to support this in US, UK and Germany.

this assumption. Harris and Pringle [8] are in the same line but allow for a continuous-time model, with possibly continuous debt rebalancing. However, they are all mute about the fact that the return on equity and the WACC will be time-dependent for finite horizon projects (even when the debt to equity ratio is kept constant, as we will show later).

Although Fernandez [5] provides examples to show that the three approaches (WACC, APV and FTE) yield the same result, Kruschwitz and Loffler [11] and Loffler [13] argue that this is wrong and that the valuation approach to be used should depend on the financing assumption, which is not true (in fact, the financing assumption can make one approach easier than the other, but all three approaches must still provide the same results, if correctly used). This debate continued and Stanton and Seasholes [20], working with perpetual cash flows, illustrated with an example in which the three methodologies provided the same valuation for a growing company with possibly risky debt and with a constant debt to equity ratio. In his turn, Sabal [18] argues that WACC and APV yield the same results as long as “cash flows are no-growth perpetuities, there is a single and constant corporate tax rate, and leverage is set as a constant proportion of the market value of the firm”.

Luehrman [14] and Lobe [12] highlight some limitations of (mis)using the traditional WACC, but neither work provides the solution to these limitation as we do in here. Farber, Gillet and Szafarz [3] discusses the adjusted present value and the classic weighted average cost of capital model, but do not comment on finite-life projects explicitly. Moreover, it focuses on the present value of the tax shield as well as Fernandez [6] and Farber, Gillet and Szafarz [4].

Massari, Roncaglio and Zanetti [15] solve the perpetual growing company problem matching the WACC and APV approaches (but notice that they are silent in respect to the return on equity and the FTE approach or in respect to finite horizon projects). Their most important result (2.8) is a particular case of a result developed here in section 2.3.2. Even classic textbooks as Jaffe, Ross and Westerfield [10] are silent about bias in finite-life projects although they stress the constant debt to equity assumption to make MM results valid.

On the good side of the debate, Sercu [19] shows that the WACC approach overestimates the NPV of projects with finite lives. Tham and Velez-Pareja [21] notice that the traditional formulation for the required return on equity only applies to perpetuities and not to finite periods. Ibragimov, Tham and Velez-Pareja [9] seem to be the first (and only) to identify that even when leverage is constant, WACC and the cost of equity will not be constant for finite-horizon cash flows. This paper bridges this gap and show exactly how the

FTE and WACC approaches must be correctly used through an easy rationale developed.

It is important to comment that it is not the concern of this paper to discuss the discount rate that tax shields should be discounted. Whatever the correct discount rate be, the analysis developed in this paper can be easily adapted. In other words, our methodology does not absolutely rely on this point.

In this paper, we make the standard assumption that the firm is always able to benefit from the tax shield in the same period as interests are paid and use the return on debt to discount the tax benefits. We indeed want to convey the methodology presented here in a very transparent way.

The paper proceeds as follows: Over the next section, we specify the problem under different assumptions and provide correct formulas to the required return on equity and to the WACC, obtained through a really intuitive and straightforward methodology. In section 3 we provide numerical examples on the valuation of a finite horizon project and of a perpetual project to show that the formulas developed before work perfectly (such that all three valuation approaches match) and that using traditional formulas may lead to quite wrong capital budgeting decisions. Section 4 concludes this paper.

## 2 Problem Specification and Solutions

We set up an investment project (that obviously nests the case of a whole firm) in discrete-time such that cash flows take place at times  $t = 0, 1, 2, \dots, T$ . The initial cash flow is  $-CF_0$ , takes place at time  $t = 0$  and the positive amount  $CF_0$  is thus the initial investment for the project (at time 0). All other cash flows are net taxes and expected to be non-negative by assumption and may (as in the real case) be stochastic: We will denote their *expected*<sup>3</sup> values at time 0 as  $CF_i$ ,  $t = 1, 2, \dots, T$ . Observe that these cash flows come strictly from operations and are commonly called in the related literature as the unlevered after-taxes cash flows. Unlevered will always refer to the project as totally financed by equity. If the project is levered, it means that it is (at least part of it) financed with debt. As usual, interests payments provide tax savings while dividends do not.

We assume that debtholders demand a return of  $R_D$  and that the investment assets of the project provide a required return equal to  $R_U$ : We recall that this is the equityholders demanded return for the unlevered project. The

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<sup>3</sup>This is only to avoid having to repeatedly write the expectation operator and keep the notation as simple as possible.

project pays income taxes at a rate  $\tau$ . For simplicity, we assume that these three parameters are constant and inputs of our model. Furthermore, we stick to the standard assumptions of perfect capital markets, no transaction costs and no agency costs.

The return equityholders require in a levered project,  $R_E$ , and the weighted average cost of capital (WACC) are outputs of the model to be calculated. Obviously, when the project is financed with no debt,  $WACC = R_E = R_U$ . The methodology carried out in this paper always departs from the unlevered project paying exactly what it should to its equityholders. In other words, the initial investment  $CF_0$  is precisely fair given the expected future cash flows and the risk of them.

## 2.1 The Single-Period Case

We begin with this simple case because it is easier to define and to first develop our methodology. Suppose a one-period all-equity investment yielding exactly what equityholders expect, that is,  $R_U$ . So we have that:

$$CF_1 = (1 + R_U) CF_0. \quad (1)$$

Now we consider the same investment levered, such that debtholders invest an amount  $D$  to receive  $D(1 + R_D)$  in one period. In this case, equityholders need to invest only  $(CF_0 - D)$ , must pay back debtholders and will benefit from tax savings due to interests paid. Therefore, equityholders will receive in one period the amount of:

$$CF_1 - D(1 + R_D) + D\tau R_D = CF_0(1 + R_U) - D(1 + R_D(1 - \tau)), \quad (2)$$

in which we have used equation 1.

Let us denote by  $V_U$  and  $V_L$  the present value of all expected future cash flows respectively of the unlevered and of the levered project<sup>4</sup>. According to the APV approach, we can relate these values as follows:

$$V_L = V_U + PV(\text{Debt Tax Benefit}) = CF_0 + \frac{D\tau R_D}{1 + R_D}, \quad (3)$$

such that equity and debtholders share the levered project value:

$$V_L = E + D \quad \rightarrow \quad E = CF_0 + \frac{D\tau R_D}{1 + R_D} - D. \quad (4)$$

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<sup>4</sup>Notice that in fact  $V_U = CF_0$ .

Notice that equityholders absorb all the tax shield benefit such that their expected return  $R_E$  on the levered project must respect the following equation:

$$CF_0(1 + R_U) - D(1 + R_D(1 - \tau)) = (1 + R_E) \left( CF_0 + \frac{D\tau R_D}{1 + R_D} - D \right). \quad (5)$$

In the equation above, we are simply stating that the cash flow to equityholders tomorrow (given by 2) must be equal to the value of this cash flow today (adjusted for the time value of money, through  $R_E$ ), since equityholders are aware that all the tax shield gains pertain to them. In other words, they will revise their (now levered) expected return taking into account the tax benefit of debt. Observe that traditionally the explanation here is that they take into account that their position is riskier now (which is also true). However, in our opinion, the first way of interpreting the revised expected return is much more straightforward and does not rely on any need for a more precise definition of risk. We continue, separating the terms  $CF_0$  and  $D$  in the previous equation to get the following:

$$CF_0(R_E - R_U) = D \left[ (1 + R_E) \left( 1 - \frac{\tau R_D}{1 + R_D} \right) - (1 + R_D(1 - \tau)) \right]. \quad (6)$$

We now use equation 4 to eliminate  $CF_0$ :

$$E + D - \frac{D\tau R_D}{1 + R_D} = D \frac{\left[ (1 + R_E) \left( 1 - \frac{\tau R_D}{1 + R_D} \right) - (1 + R_D(1 - \tau)) \right]}{(R_E - R_U)}. \quad (7)$$

Dividing both sides by the equity value and isolating the debt-to-equity ratio, we obtain:

$$1 = \frac{D}{E} \left\{ \frac{\left[ (1 + R_E) \left( 1 - \frac{\tau R_D}{1 + R_D} \right) - (1 + R_D(1 - \tau)) \right]}{(R_E - R_U)} + \frac{\tau R_D}{1 + R_D} - 1 \right\}, \quad (8)$$

which after some algebra can be written as follows:

$$\boxed{R_{E,T=1} = R_U + \frac{D}{E} (R_U - R_D) \left( 1 - \tau + \frac{\tau}{1 + R_D} \right)}, \quad (9)$$

in which we have written  $R_{E,T=1}$  to denote this is the expected return on equity for projects with 1 period to maturity.

Notice that this result is different from the classic one obtained by Modigliani and Miller [17] precisely due to the last term. This is because in their paper they consider infinite cash flows and hence a much greater tax shield. When

they added taxes to their model, the impact on the expected return on equity was the factor  $(1 - \tau)$  multiplying the leverage slope, meaning that the tax shield was decreasing the levered project risk. Here, as the tax shield lasts for only one period, its impact in terms of decreasing the levered project risk is lower. Notice that if there are no interests ( $R_D = 0$ ), there is no tax gain and we recover the Modigliani and Miller [16] result (*i.e.*, without taxes).

And what about the WACC? By definition, the WACC should be the rate that discounting the *unlevered* cash flow(s) provides the correct value of the project. So it must be such that the identity below holds:

$$V_L = \frac{CF_1}{1 + WACC} = CF_0 + \frac{D\tau R_D}{1 + R_D}. \quad (10)$$

We can use equation 1 and the fact that  $D = \frac{D}{(D+E)} (D + E) = \frac{D}{(D+E)} \frac{CF_1}{(1+WACC)}$  to write down the following:

$$\frac{1}{1 + WACC} = \frac{1}{1 + R_U} + \frac{D}{(D + E)} \frac{\tau R_D}{(1 + R_D)(1 + WACC)}, \quad (11)$$

which solving for the WACC gives:

$$WACC_{T=1} = R_U \left[ 1 - \frac{D}{(D + E)} \tau \frac{(1 + R_U) R_D}{(1 + R_D) R_U} \right], \quad (12)$$

in which we have written  $WACC_{T=1}$  to denote this is the weighted average cost of capital for projects with 1 period to maturity.

We can now solve equation 12 for  $R_U$  and plug the result into equation 9. Very tedious calculation will provide the formula for the weighted average cost of capital in the one-period project case:

$$\boxed{WACC_{T=1} = \frac{E}{D + E} R_E + \frac{D}{D + E} R_D (1 - \tau)}, \quad (13)$$

which is exactly the textbook formulation for this discount rate: This is the case because the tax benefit in period 1 is still given by  $D\tau R_D$ , as in MM world. So the main lesson we take from the one-period project is that one needs to reformulate the return on equity for the levered projects. As such, using the traditional textbook formula, the FTE approach will provide wrong valuation as well as the WACC approach since  $R_E$  is an input for the WACC. But we stress that this is not a problem with the methodology: Using the correct formulas that we showed above will make any of the three valuation approaches (APV, FTE and WACC) to match, as it must be the case since they all measure the same project value.

## 2.2 The Multi-Period Case

For a project with more periods, one important issue is to define the debt policy for the project. We begin with the standard case of a constant debt level and analyze other cases in the sequence.

### 2.2.1 Constant Debt Level

Suppose an  $n$ -period all-equity investment yielding exactly what equityholders expect, that is,  $R_U$ . The expected cash flow is then the following:

$$CF_n = (1 + R_U)CF_0 \quad \text{and} \quad CF_i = R_U CF_0 \quad \text{for } i = 1, 2, \dots, (n - 1). \quad (14)$$

Now we consider the same investment levered, such that debtholders invest an amount  $D$  to receive  $DR_D$  every period, except in the last one, in which they receive interests plus the principal back:  $D(1 + R_D)$ . On their turn, equityholders invest only  $(CF_0 - D)$  to receive in every period<sup>5</sup> the amount of  $R_U CF_0 - DR_D(1 - \tau)$ .

Remember that the project value (calculated through the APV approach) is:

$$V_L = V_U + PV(\text{Debt Tax Benefit}), \text{ or} \quad (15a)$$

$$V_L = CF_0 + \sum_{i=1}^n \frac{D\tau R_D}{(1 + R_D)^i} = CF_0 + D\tau \left(1 - \frac{1}{(1 + R_D)^n}\right). \quad (15b)$$

The logic we follow now is exactly the same as explained in the single-period project. The cash flow to equityholders at  $t = 1$  plus their share of the project at  $t = 1$  must match the value of equity at  $t = 0$  adjusted for their expected return on the levered project  $R_E$ . Therefore, the expected return  $R_E$  for equityholders is such that the equation below must hold:

$$\begin{aligned} & CF_0 R_U - DR_D(1 - \tau) + CF_0 + D\tau \left(1 - \frac{1}{(1 + R_D)^{n-1}}\right) - D = \\ & = (1 + R_E) \left( CF_0 + D\tau \left(1 - \frac{1}{(1 + R_D)^n}\right) - D \right). \end{aligned} \quad (16)$$

We can then write the following (notice that  $V_L$ ,  $E$  and  $D$  refer to the respective values at time 0):

$$V_L = E + D = CF_0 + D\tau \left(1 - \frac{1}{(1 + R_D)^n}\right), \quad (17a)$$

<sup>5</sup>Except in the last period, in which equityholders receive the same as in the single-period case previously analysed.



$$CF_0 = E + D - D\tau \left( 1 - \frac{1}{(1 + R_D)^n} \right). \tag{17b}$$

We do some tedious algebra with equation 16, first separating the terms  $CF_0$  and  $D$ , then using equation 17b and rearranging terms to obtain the following result:

$$R_{E,T=n} = R_U + \frac{D}{E} (R_U - R_D) \left( 1 - \tau + \frac{\tau}{(1 + R_D)^n} \right), \tag{18}$$

in which we have been consistent to the notation of the single-period project, and express that the expected return on equity will depend on the number of periods that the project will still last.

As far as we are concerned, this result is new in the literature. Notice that the single-period result is a particular case of the formula above, as well as Modigliani and Miller [17] classic result for the infinite horizon ( $n \rightarrow \infty$ ). In fact the result above generalizes both the one-period project analyzed before and MM’s result. As the number of periods increase, the tax shield benefits also increase, decreasing equityholders risk, reflecting thus a lower discount rate.

The relationship given by 18 should be interpreted as follows: When the debt level is held constant and the project has a finite horizon of existence,  $R_E$  (and consequently the WACC) change every period because they depend on how many periods the project will last. The condition of a constant debt level is needed due to the way the present value of tax shield benefits was calculated above.

Proceeding to the WACC, this rate must be such that the identity below holds:

$$V_L = \frac{CF_1 + V_{L, \text{next period}}}{1 + WACC}, \tag{19a}$$

which can be rewritten as:

$$CF_0 + D\tau \left( 1 - \frac{1}{(1 + R_D)^n} \right) = \frac{R_U}{1 + WACC} CF_0 + \frac{CF_0 + D\tau \left( 1 - \frac{1}{(1 + R_D)^{n-1}} \right)}{1 + WACC}. \tag{19b}$$

Some boring algebra yields:

$$WACC_{T=n} = R_U \left[ 1 - \frac{D}{(D + E)} \tau \left( 1 + \frac{\frac{R_D}{R_U} - 1}{(1 + R_D)^n} \right) \right]. \tag{20}$$

Observe that this equation nests both single-period and Modigliani and Miller [17] (infinite horizon) cases too. We can now solve equation 20 for  $R_U$

and plug the result into equation 18. Very tedious calculation will provide the formula for the weighted average cost of capital in the multi-period project:

$$\boxed{WACC_{T=n} = \frac{E}{D+E}R_E + \frac{D}{D+E}R_D(1-\tau)}, \quad (21)$$

which is exactly the textbook formulation for this discount rate but we stress out that the WACC is period dependent and no longer constant not only because the leverage ratio is likely to change in the project but also because the return on equity will change from period to period.

The main lesson we take from the multi-period project with a constant debt level is that one needs to reformulate the return on equity for levered projects and this rate will be time-varying even when the debt to equity ratio remains constant! As such, using the traditional textbook formula, the FTE approach will provide wrong valuation as well as the WACC approach since  $R_E$  is an input for the WACC. But, as in the one-period case, we reaffirm that this is not a problem with the methodology: Using the correct formula for the expected return on equity will make any of the three valuation approaches (APV, FTE and WACC) to match, as we will show in section 3.

### 2.2.2 Debt Repaid With Constant Amortization

This really seems to be a forgotten case in the literature. It is not uncommon to repay the debt used to finance (part of) a project with constant amortization, such that the updated value of debt drops linearly as periods go by. To develop our method, we suppose the same  $n$ -period all-equity investment as before and then include an initial amount  $D$  of debt at a cost of  $R_D$ . However now debtholders receive periodically not only interests but also a constant amortization of the debt:  $D\frac{n+1-i}{n}R_D + \frac{D}{n}$  in every period  $i = 1, 2, \dots, n$ . On their turn, equityholders invest only  $(CF_0 - D)$  to receive in every period the amount of  $R_U CF_0 - D\frac{n+1-i}{n}R_D(1-\tau) - \frac{D}{n}$  (plus  $CF_0$  in the last period).

Remember that the project value (calculated through the APV approach) is:

$$V_L = V_U + PV(\text{Debt Tax Benefit}), \text{ or} \quad (22a)$$

$$V_L = CF_0 + D\tau R_D \sum_{i=1}^n \frac{\frac{n+1-i}{n}}{(1+R_D)^i}. \quad (22b)$$

The equality above can be simplified to:

$$V_L = CF_0 + D\tau \frac{(nR_D - 1)(1+R_D)^n + 1}{nR_D(1+R_D)^n}, \text{ or} \quad (22c)$$

$$V_L = CF_0 + D\tau K_n, \tag{22d}$$

in which we had conveniently defined the quantity  $K_n$ .

The cash flow to equityholders at  $t = 1$  plus their share of the project at  $t = 1$  must match the value of equity at  $t = 0$  adjusted for their expected return on the levered project  $R_E$ . Therefore, the expected return  $R_E$  for equityholders is such that the equation below must hold:

$$\begin{aligned} R_U CF_0 - DR_D(1 - \tau) + CF_0 + D\frac{n-1}{n}\tau K_{n-1} - D &= \\ &= (1 + R_E)(CF_0 + D\tau K_n - D). \end{aligned} \tag{23}$$

which can be written as:

$$D \left[ \frac{n-1}{n}\tau K_{n-1} - R_D(1 - \tau) - (1 + R_E)\tau K_n + R_E \right] = (R_E - R_U)CF_0. \tag{24}$$

We can write the following (notice that  $E$  refers to the equity value at time 0):

$$V_L = E + D = CF_0 + D\tau K_n \quad \rightarrow \quad CF_0 = E + D - D\tau K_n. \tag{25}$$

We do some tedious algebra with equations 24 and 25 to obtain the following result:

$$R_{E,T=n} = R_U + \frac{D}{E} \left[ R_U(1 - \tau K_n) - \tau \frac{(1 + R_D)^n - 1}{n(1 + R_D)^n} - R_D(1 - \tau) \right], \tag{26}$$

in which we have been consistent and expressed the expected return on equity as depending on the number of periods that the project will still last.

Proceeding to the WACC, this rate must be such that the identity below holds:

$$V_L = \frac{CF_1 + V_{L, \text{next period}}}{1 + WACC}, \tag{27a}$$

$$CF_0 + D\tau K_n = \frac{R_U}{1 + WACC}CF_0 + \frac{CF_0 + D\tau K_n}{1 + WACC}. \tag{27b}$$

Some boring algebra yields:

$$WACC_{T=n} = R_U \left( 1 - \frac{D}{(D + E)}\tau K_n \right) - \frac{D}{(D + E)}\tau \frac{(1 + R_D)^n - nR_D - 1}{n(n - 1)R_D(1 + R_D)^n}. \tag{28}$$

We can now solve equation 28 for  $R_U$  and plug the result into equation 26. Very tedious calculation will provide the formula for the weighted average cost of capital in the multi-period project:

$$\boxed{WACC_{T=n} = \frac{E}{D+E}R_E + \frac{D}{D+E}R_D(1-\tau)}, \quad (29)$$

which is the usual formula. But notice that equation 26 is completely new in the literature and again the FTE and WACC approaches, if not using this correct formula, will provide wrong valuations.

### 2.2.3 A Final Comment to the Finite Horizon Case

We would like to end this subsection highlighting that the methodology to correctly calculate  $R_E$  and therefore WACC could be applied virtually to any other repayment schemes as for example the common cases of assuming constant debt-to-equity ratio or constant payments to debtholders. However the algebra gets easily involved and the final expression to  $R_E$  is no longer sympathetic. In what concern the expression for WACC based on  $R_E$  and  $R_D$ , the standard formula will always be true as long as all interests due are paid in every period (providing benefits from the full debt shield as a consequence). The next section will show an example in which this condition does not hold, causing the WACC final expression to be different than the standard one.

Furthermore, when the leverage ratio is constant, some authors<sup>6</sup> argue that the correct discount rate to the debt shield is no longer  $R_D$ : Since debt becomes proportional to equity, its tax shield benefits should thus be considered riskier. While the author of this paper agrees with this idea, it is not a concern of this paper to discuss appropriate discount rates, but to develop a rationale to get correct formulas to the required return on equity and as a consequence to the weighted average cost of capital in any situation. The discount rates used of course will impact the final results, however the rationale presented here will not change at all.

## 2.3 The Perpetuity Case with Growth

Suppose a perpetuity all-equity project with initial investment  $-CF_0$  yielding exactly what equityholders expect, that is,  $R_U$  such that the unlevered project value is expected to grow at a given rate  $g$ . The value of the unlevered

<sup>6</sup>For example, see Ezzell and Miles [2] and Fernandez [7].

project at any period  $i$  is  $V_{U,i}$ , in which  $V_{U,i} = (1 + g) V_{U,i-1}$ . The cash flow proposed is therefore the following:

$$CF_i = (R_U - g) V_{U,i-1}, \quad i = 1, 2, \dots \quad (30)$$

### 2.3.1 Constant Debt to Equity Ratio

Now we consider the same project but levered, such that debtholders hold a constant fraction of the project, that is,  $D_i/V_i$ , is kept constant and equal to  $D_0/V_{L,0}$ . On their turn, equityholders invest initially only  $(CF_0 - D_0)$ . Every period, debtholders will receive  $(R_D - g) D_{i-1}$ . Notice that the debt also increases at the rate  $g$  to guarantee that the debt-to-equity ratio is held constant:  $D_i = (1 + g) D_{i-1}$ . The periodic cash flow to equityholders is  $(R_U - g) V_{U,i-1} - (1 - \tau) (R_D - g) D_{i-1}$ .

Remember that the project value at time 0 is:

$$V_{L,0} = V_{U,0} + PV(\text{Debt Tax Benefit}), \text{ or} \quad (31a)$$

$$V_{L,0} = CF_0 + \sum_{j=1}^{\infty} \frac{\tau (R_D - g) D_{j-1}}{(1 + R_D)^j} = CF_0 + D_0 \tau. \quad (31b)$$

The expected return  $R_E$  for equityholders at any period  $i$  is such that the equation below must hold:

$$(R_U - g) V_{U,i-1} - (1 - \tau) (R_D - g) D_{i-1} + (V_{L,i} - D_i) = (1 + R_E) (V_{L,i-1} - D_{i-1}), \quad (32a)$$

which applied for period 1 gives:

$$(R_U - g) CF_0 - (1 - \tau) (R_D - g) D_0 = (R_E - g) (V_{L,0} - D_0). \quad (32b)$$

We can develop the previous relationship further to obtain:

$$\boxed{R_E = R_U + \frac{D}{E} (R_U - R_D) (1 - \tau)}. \quad (33)$$

Since the debt-to-equity ratio in the project is constant, we can just omit the subscripts and conclude that the expected return on equity will be constant. Now let us analyze in terms of WACC. It must be such that the equation below is valid:

$$V_{L,0} = \frac{CF_1 + V_{L,1}}{1 + WACC}, \quad (34)$$

Some boring algebra yields:

$$WACC = R_U \left[ 1 - \frac{D}{E + D} \tau \left( 1 - \frac{g}{R_U} \right) \right], \quad (35)$$

in which again we omitted the time subscripts because the leverage ratio is constant.

We can now solve equation 35 for  $R_U$  and plug the result into equation 33. Very tedious calculation will provide the formula for the weighted average cost of capital in the perpetuity case with growth:

$$\boxed{WACC = \frac{E}{E+D}R_E + \frac{D}{E+D}[(1-\tau)R_D + \tau g]}, \quad (36)$$

which is different from the usual formula. The different WACC formula is due to the fact that in the standard approach, it is implicitly assumed that all interests will be paid and hence the tax benefits will apply over all theoretical interests. Here, if we assume the project will perpetually grow at a constant rate, with equity and debt growing at the same rate to keep the debt-to-equity ratio constant, not all the interests are paid (some are capitalized, increasing the debt): Therefore, the debt tax shield needs to be adjusted accordingly. It is interesting to point out that if on the one hand the return on equity, as given by 33, is indeed the same as in MM world (i.e., with no growth), on the other hand this is no longer true to the weighted average cost of capital! In other words, growth will make MM classic results not valid.

### 2.3.2 Interests Fully Repaid and New Debt Issued to Keep D/E Constant

If we consider that the interests are fully repaid each period (for taxes purposes) and the debt increase is achieved with new debt issuance, the rationale above can be repeated leading to the following result:

$$\boxed{R_E = R_U + \frac{D}{E} \left[ R_U \left( 1 - \tau \frac{R_D}{R_D - g} \right) + g\tau \frac{R_D}{R_D - g} - R_D(1 - \tau) \right]}. \quad (37)$$

Notice that the expected return on equity differs from the previous case such that growth makes again MM results not valid. The following relationships will also be true:

$$WACC = R_U \left[ 1 - \frac{D}{E+D} \tau \frac{R_D}{R_D - g} \left( 1 - \frac{g}{R_U} \right) \right], \quad (38)$$

and

$$\boxed{WACC = \frac{E}{E+D}R_E + \frac{D}{E+D}R_D(1 - \tau)}, \quad (39)$$

from which we can see that when interests are fully paid every period (again, for taxes purposes), the standard formula for the WACC will hold.

### 2.3.3 The Case of Constant Debt

If, instead of a constant debt to equity ratio, we consider that debt is kept constant, one can easily show that the familiar textbook formulas below are indeed correct:

$$R_E = R_U + \frac{D}{E} (R_U - R_D) (1 - \tau), \quad (40)$$

$$WACC = R_U \left( 1 - \frac{D}{E + D} \tau \right), \quad (41)$$

$$WACC = \frac{E}{E + D} R_E + \frac{D}{E + D} R_D (1 - \tau). \quad (42)$$

However, this does not mean that traditional FTE and WACC valuations will be correct! This comes from the fact that, unlike in the previous case, the debt to equity ratio does not remain constant. The problem arises because to evaluate the perpetuity, one uses the well known formula that divides the next period cash flow divided by the rate minus the growth rate. And, as  $D/E$  is not constant, so do the rates  $R_E$  and WACC such that this formula will provide wrong values. In other words, one would have to stick to the APV formulation or get to quite involved formulas to be able to use the FTE or WACC approaches. In the next section, we provide a numerical example to show that the valuation error can be considerable.

Finally, just before we proceed to the application section, we point out that in all results above, if  $g = 0$  (i.e., no growth) then one obtains the original MM formulas, which should come with no surprise since in such case we meet all their assumptions. In other words, MM classic results are a very particular case of ours.

## 3 Application of the Results

In this section, we illustrate some numerical examples to apply the results previously developed and show that they indeed work and, perhaps most importantly, all three fundamental valuation approaches (WACC, APV and FTE) will converge to the same project value. We will also show that using traditional formulas may lead to sizeable valuation errors and wrong capital budgeting decisions.

For all the following examples, we consider that the cost of debt is  $R_D = 8\%$ , the required return to the unlevered project is  $R_U = 16\%$  and that taxes are 40%.

### 3.1 Valuation of a Finite Horizon Project

Suppose a 4-year project with a total \$1,000 initial investment, from which \$400 will be financed through debt. The expected after-taxes cash flows for the unlevered project are respectively \$200, \$300, \$400 and \$540 for years 1, 2, 3 and 4. We now consider two different cases, for illustration purposes.

#### 3.1.1 Principal is Fully Repaid at the End of Project

In such case, only interests are paid in periods 1, 2 and 3 and debt is kept therefore constant at \$400. Using traditional (and wrong) formulas (which are given by 40 and 42), one would calculate the following discount rates:  $R_E = 19.24\%$  and  $WACC = 13.42\%$ . With these rates, the FTE approach would provide a valuation of \$33.46 and the WACC approach, a value of \$10.01, while the APV approach would generate the correct valuation of  $-\$7.74$ . This means that a manager using the textbook formulas for the return on equity and for the WACC would incorrectly accept the levered project, when it should not since its true NPV is negative. Even if the manager is aware that the debt to equity ratio is not constant and adjusts period by period both  $R_E$  and the WACC, it will find misleading results (taking again the wrong decision of accepting the project either using FTE or WACC approaches), as indicated in figure 1.

**Figure 1:** Even adjusting the traditional textbook formulas for a time-varying debt to equity ratio, the FTE and WACC valuation approaches will be misleading because those formulas are incorrect.

Year	After-Tax Cash Flows	Face Value Debt	Tax Shield	After-Tax Equity CF's	PV of Future CF's	D/E	$R_E$	Equity PV	WACC	Unlevered CF PV	
0	-\$1,000.00	\$400.00	\$0.00	-\$600.00	\$992.26	0.68	--x--	-\$600.00	--x--	-\$1,000.00	
1	\$200.00	\$400.00	\$12.80	\$180.80	\$934.83	0.75	19.24%	\$151.62	13.42%	\$176.34	
2	\$300.00	\$400.00	\$12.80	\$280.80	\$768.96	1.08	19.59%	\$196.91	13.26%	\$233.53	
3	\$400.00	\$400.00	\$12.80	\$380.80	\$477.37	5.17	21.20%	\$220.32	12.67%	\$276.36	
4	\$540.00	\$0.00	\$12.80	\$120.80	--x--	--x--	40.82%	\$49.63	10.64%	\$337.22	
<b>All-Equity Firm</b>		<b>Levered Firm - APV Approach</b>				<b>Levered Firm - FTE Approach</b>		<b>Levered Firm - WACC Approach</b>			
NPV	-\$50.14	APV				NPV		\$18.49	NPV		\$23.45

We can conclude from this example that the error committed when using the textbooks formulas for either the FTE or the WACC approach is sizeable. In figure 2, we present the values when correctly calculated according to the formulas developed in section 2.2.1 and as a result we can see that all approaches will provide exactly the same valuation, thus leading to the same (correct) decision. For each period, equation 18 was used to calculate the required return on equity.



**Figure 2:** When using correct formulas for the return on equity and WACC, the FTE and WACC valuation approaches will provide correct valuations.

Year	After-Tax Cash Flows	Face Value Debt	Tax Shield	After-Tax Equity CF's	PV of Future CF's	D/E	$R_E$	Equity PV	WACC	Unlevered CF PV
0	-\$1,000.00	\$400.00	\$0.00	-\$600.00	\$992.26	0.68	--x--	-\$600.00	--x--	-\$1,000.00
1	\$200.00	\$400.00	\$12.80	\$180.80	\$934.83	0.75	20.83%	\$149.63	14.37%	\$174.87
2	\$300.00	\$400.00	\$12.80	\$280.80	\$768.96	1.08	21.49%	\$191.29	14.35%	\$229.40
3	\$400.00	\$400.00	\$12.80	\$380.80	\$477.37	5.17	24.18%	\$208.90	14.10%	\$268.07
4	\$540.00	\$0.00	\$12.80	\$120.80	--x--	--x--	56.13%	\$42.44	13.12%	\$319.92
<b>All-Equity Firm</b>		<b>Levered Firm - APV Approach</b>					<b>Levered Firm - FTE Approach</b>		<b>Levered Firm - WACC Approach</b>	
NPV	-\$50.14	APV				NPV	\$7.74	NPV	-\$7.74	

### 3.1.2 Principal is Paid with Constant Amortization

Here we assume that debt is constantly amortized and hence in each period full interests plus  $\frac{\$400}{4} = \$100$  of principal amortization is paid back to debtholders such that the debt level decreases linearly. Using traditional formulas, one calculates the following discount rates:  $R_E = 19.33\%$  and  $WACC = 13.38\%$ . With these rates, the FTE approach would provide a valuation of  $-\$16.35$  and the WACC approach, a value of  $\$10.97$ , while the APV approach would generate the correct valuation of  $-\$22.62$ . Notice that the project value decreases because now debt is repaid faster and therefore the tax shield generates less value than in the previous case. Even if the manager is aware that the debt to equity ratio is not constant and adjusts period by period both  $R_E$  and the WACC, it will find misleading results, as indicated in figure 3.

**Figure 3:** Even adjusting the traditional textbook formulas for a time-varying debt to equity ratio, the FTE and WACC valuation approaches will be misleading because those formulas are incorrect.

Year	After-Tax Cash Flows	Face Value Debt	Tax Shield	After-Tax Equity CF's	PV of Future CF's	D/E	$R_E$	Equity PV	WACC	Unlevered CF PV
0	-\$1,000.00	\$400.00	\$0.00	-\$600.00	\$977.38	0.69	--x--	-\$600.00	--x--	-\$1,000.00
1	\$200.00	\$300.00	\$12.80	\$80.80	\$918.76	0.48	19.33%	\$67.71	13.38%	\$176.40
2	\$300.00	\$200.00	\$9.60	\$185.60	\$754.80	0.36	18.33%	\$131.45	13.91%	\$232.28
3	\$400.00	\$100.00	\$6.40	\$290.40	\$468.48	0.27	17.73%	\$174.70	14.30%	\$270.95
4	\$540.00	\$0.00	\$3.20	\$435.20	--x--	--x--	17.30%	\$223.19	14.63%	\$319.09
<b>All-Equity Firm</b>		<b>Levered Firm - APV Approach</b>					<b>Levered Firm - FTE Approach</b>		<b>Levered Firm - WACC Approach</b>	
NPV	-\$50.14	APV				NPV	-\$2.95	NPV	-\$1.27	

In figure 4, we present the values when correctly calculated according to the formulas developed in section 2.2.2 and as a result we can see that all approaches will provide exactly the same valuation, as it must be the case. For each period, equation 26 was used to calculate the required return on equity.

**Figure 4:** When using correct formulas for the return on equity and WACC, the FTE and WACC valuation approaches will provide correct valuations.

Year	After-Tax Cash Flows	Face Value Debt	Tax Shield	After-Tax Equity CF's	PV of Future CF's	D/E	$R_E$	Equity PV	WACC	Unlevered CF PV
0	-\$1,000.00	\$400.00	\$0.00	-\$600.00	\$977.38	0.69	--x--	-\$600.00	--x--	-\$1,000.00
1	\$200.00	\$300.00	\$12.80	\$80.80	\$918.76	0.48	21.16%	\$66.69	14.47%	\$174.73
2	\$300.00	\$200.00	\$9.60	\$185.60	\$754.80	0.36	19.66%	\$128.02	14.81%	\$228.28
3	\$400.00	\$100.00	\$6.40	\$290.40	\$468.48	0.27	18.76%	\$168.66	15.06%	\$264.54
4	\$540.00	\$0.00	\$3.20	\$435.20	--x--	--x--	18.11%	\$214.01	15.27%	\$309.83
<b>All-Equity Firm</b>		<b>Levered Firm - APV Approach</b>					<b>Levered Firm - FTE Approach</b>		<b>Levered Firm - WACC Approach</b>	
NPV	-\$50.14	APV					NPV	-\$22.62	NPV	-\$22.62

## 3.2 Valuation of a Perpetual Growing Project

Suppose an infinite horizon project with a total \$1,000 initial investment, from which \$400 will be (initially) financed through debt. The expected after-taxes cash flows for the unlevered project are expected to grow at a 4% rate and the next period cash flow is estimated at \$100.

### 3.2.1 Constant Debt to Equity Ratio with Interests Added to Principal

As we have discussed before, debt will increase with non-paid interests to maintain constant the capital structure. The correct valuation by APV provides a valuation<sup>7</sup> of  $-\$6.67$  (so that the project should be rejected in such terms). We have seen that in this case the required return on equity is given by the traditional formula and therefore the FTE approach will indeed provide the same valuation. However, the WACC formula must be adjusted under these conditions. If a manager uses the traditional formula, he or she will find 13.42% which applied to the perpetuity will give a valuation of \$61.25: Observe that this manager would erroneously accept the project! If he or she is aware that the WACC must be adjusted by equation 36, a correct value of 14.07% should be calculated, providing thus the correct valuation of  $-\$6.67$  to this perpetual project.

### 3.2.2 Constant Debt to Equity Ratio with Interests Fully Paid

In such case, new debt is issued every period to keep the capital structure constant. As already discussed, this is an important point because it will impact the tax shield and therefore the correct formulas for  $R_E$  and for the WACC. The perpetuity under the textbook formulas will provide valuations of \$65.35 and \$22.46 using respectively the FTE and the WACC approaches,

<sup>7</sup>We had used the cost of debt to discount the tax shield and to find the reported value.

while the correct value is in fact much larger: \$153.33. Figure 5 provides the correct values using the adjusted formula for  $R_E$  as in 37. As a consequence, we obtain correct valuations under any of the approaches.

**Figure 5:** When using correct formulas for the return on equity and WACC, the FTE and WACC valuation approaches will provide correct valuations.

Year	After-Tax Cash Flows	Face Value Debt	Tax Shield	After-Tax Equity CF's	PV of Future CF's	D/E	$R_E$	Equity PV	WACC	Unlevered CF PV
0	-\$1,000.00	\$400.00	\$0.00	-\$600.00	\$1,153.33	0.53	--x--	-\$600.00	--x--	-\$1,000.00
1	\$100.00	\$416.00	\$12.80	\$96.80	\$1,199.47	0.53	16.85%	\$82.84	12.67%	\$88.75
2	\$104.00	\$432.64	\$13.31	\$100.67	\$1,247.45	0.53	16.85%	\$73.73	12.67%	\$81.92
...	...	...	...	...	...	...	...	...	...	...
<b>All-Equity Firm</b>		<b>Levered Firm - APV Approach</b>				<b>Levered Firm - FTE Approach</b>		<b>Levered Firm - WACC Approach</b>		
NPV	-\$166.67	APV	\$153.33		NPV	\$153.33	NPV	\$153.33		

### 3.2.3 Constant Debt

We have shown that in the perpetual growth case with constant debt, the textbook formulas for  $R_E$  and WACC are correct, however these rates are not constant and as a consequence the traditional perpetuity formulas cannot be used. To give an example of how sizeable can be the error, we apply in this example (debt is kept constant at \$400) the perpetuity formula for both the FTE and WACC approaches and compare them to the correct valuation as given by the APV approach.

The correct valuation of the project analyzed in this section shows that it is worth \$153.33. The perpetuity valuation under the FTE approach would wrongly value the project as  $-\$69.68$ , and therefore managers relying in this misapplied approach would take the wrong decision of not accepting the project! On its turn, the WACC approach will underestimate the project value as \$61.25, which means a mispricing of 60%!

## 4 Conclusion

This paper proposes a simple methodology that calculates the correct values for the expected return on equity and for the WACC of a given levered project, based on its assumptions about the debt repayment strategy. We showed that this methodology makes the WACC, the FTE and the APV approaches to perfectly match. Although the latter can be a lot easier under some circumstances, practitioners still prefer to use the WACC and, especially when the company is deeply in debt, the FTE approach: this empirical fact explains the importance of using both WACC and FTE appropriately.

A very important result is the fact that the expected return on equity for finite horizon projects will be horizon dependent, which means that it will (sometimes drastically) change as the project life shortens even when all other variables remain constant (e.g., the leverage ratio). Obviously, this result will directly impact the WACC since the return on equity is one of the weighted costs. Another interesting result is that the traditional textbook formula for the WACC is not always correct, even for perpetual projects. We have shown that if part of the interests are capitalized (i.e., not paid back to debtholders), the WACC formula needs to be adjusted in a way the literature has completely missed up to now. And this seems to be a realistic case in some growing projects in which debt will grow together with the project itself. Even in the simple case of a constant debt level under constant and perpetual growth, in which textbook formulas for the return on equity and for the WACC are indeed correct, the traditional perpetuity valuation will no longer be correct because both  $R_E$  and WACC will not be held constant!

We do believe that managers and entrepreneurs will find the results demonstrated in this paper useful not only to update their valuation techniques and therefore to be able to take correct capital budgeting decisions but also to evaluate their investments and their costs of capital more appropriately.

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